Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MM04

Unit Mechanics 4

Friday 21 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

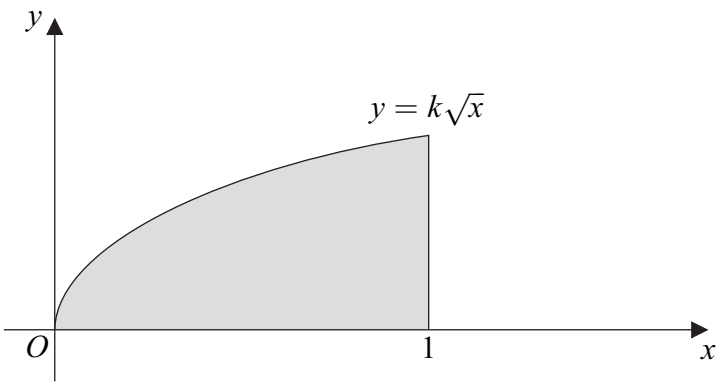
Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 A uniform lamina is bounded by the positive x-axis, the line $x = 1$ and the curve with equation $y = k\sqrt{x}$, where $k > 0$. The lamina is shown in the diagram.



The area of the lamina is A square units.

(a) Show that the x-coordinate of the centre of mass of the lamina is $\frac{2k}{5A}$. *(3 marks)*

(b) Find, in terms of k and A, the y-coordinate of the centre of mass of the lamina. *(3 marks)*

(c) Given that the centre of mass of the lamina lies on the line $y = x$, find the value of k. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

2 Three forces, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ a \end{bmatrix}$ newtons, act at the points with coordinates $(3, 4)$, $(-2, -3)$ and $(2, 0)$ metres respectively.

This system of three forces is equivalent to a force, $\mathbf{F}$, acting at the origin together with a couple of magnitude 24 Nm.

(a) Determine the two possible values for a. *(6 marks)*

(b) Write down the force, $\mathbf{F}$, corresponding to each value of a. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 2

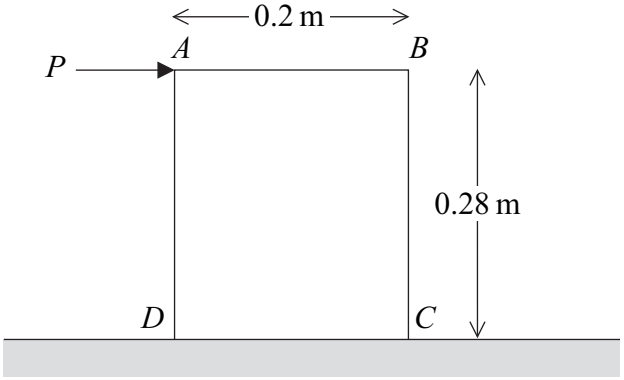
Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

3 A box of breakfast cereal, of total mass 0.4 kg, rests on a rough horizontal plane. It can be assumed that the centre of the mass of the box with its contents is on a vertical line through the centre of the box. The diagram shows a vertical cross section $ABCD$ through the centre of mass of the box and its contents, where $AB = 0.2\,\text{m}$ and $BC = 0.28\,\text{m}$. A horizontal force of magnitude P is applied at A, in the direction of AB.



The coefficient of friction between the box and the plane is μ. As P is gradually increased from zero, the box slides before it topples if, and only if, $\mu < k$. Find the value of k. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE

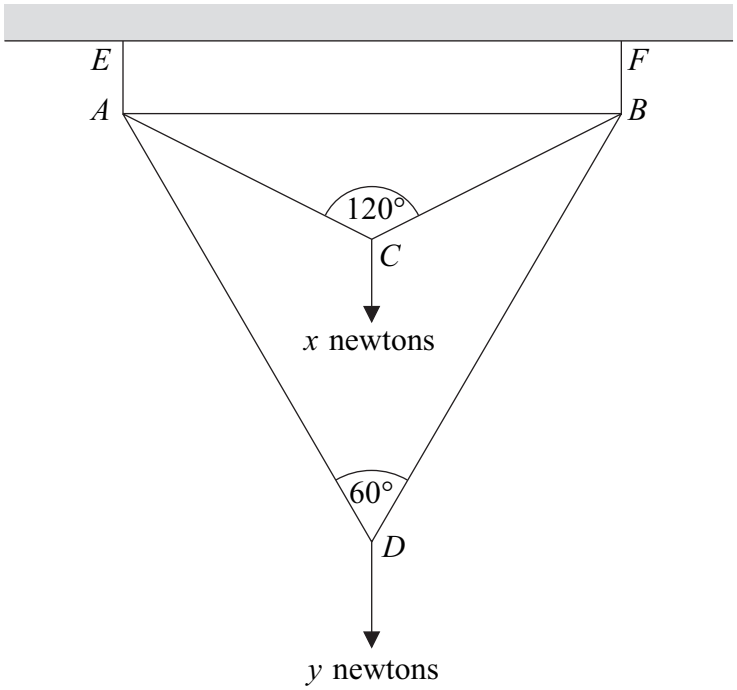
Answer space for question 3

Do not write outside the box

4 A framework $ABCD$ consists of five light rods AB, AC, BC, AD and BD, which are smoothly jointed at A, B, C and D. The angle ACB is 120° and the angle ADB is 60°.

The framework is suspended from the fixed points E and F by strings EA and FB respectively. The rod AB is horizontal and D is vertically below C. The framework has a line of symmetry through C and D.

A force of x newtons is applied vertically downwards at C and a force of y newtons is applied vertically downwards at D. The system is in equilibrium.



The forces in the rods AC, AD, BC and BD are equal in magnitude.

(a) Show that $x : y = 1 : \sqrt{3}$. *(4 marks)*

(b) The tensions in the strings EA and FB are both 100 N.

(i) Find the magnitude of the force in rod AC. *(3 marks)*

(ii) Find the magnitude of the force in rod AB, stating whether the rod is in tension or compression. *(3 marks)*

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

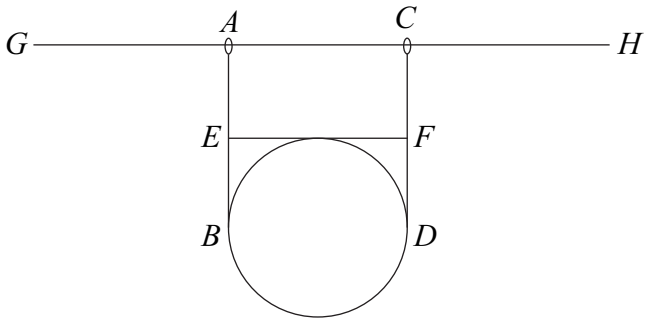
Do not write outside the box

5 (a) A uniform circular disc has mass $20m$ and radius a.

(i) Prove, by integration, that the moment of inertia of the disc about an axis through its centre and perpendicular to the plane of the disc is $10ma^2$. *(5 marks)*

(ii) Hence determine the moment of inertia of the disc about a diameter, stating clearly any theorems that you use. *(3 marks)*

(b) A shop sign consists of a uniform circular disc of mass $20m$ and radius a, which is rigidly fixed to three rods AB, CD and EF. Each rod is of mass $2m$ and length $2a$. The rods are attached so that BD is a diameter of the disc; EF is a tangent to the disc, with A vertically above E and C vertically above F. The sign is suspended on a fixed horizontal pole GH by means of two small light rings which are attached to the sign at A and C, as shown in the diagram.



The sign can swing freely about the pole GH.

Find, in terms of m and a, the moment of inertia of the sign about GH. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

6 Four forces act at the points with coordinates as listed below.

Force		Point
$2\mathbf{i} + 3\mathbf{j}$	acts at	$A(1, 0, 0)$
$2\mathbf{i} - 3\mathbf{j}$	acts at	$B(1, 1, 0)$
$-\mathbf{j} + 2\mathbf{k}$	acts at	$C(0, 0, 1)$
$\mathbf{j} + 2\mathbf{k}$	acts at	$D(0, 1, 1)$

(a) Find the resultant of the four forces. *(1 mark)*

(b) Find the total moment of the four forces about O. *(5 marks)*

(c) An additional force **P** acts at a point on the y-axis so that the five forces are in equilibrium.

Determine the coordinates of the point on the y-axis at which **P** acts. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

7 A system consists of a uniform rod AB, of mass $3m$ and length $8a$, and a particle of mass m attached to the end B. The system can rotate freely about a fixed horizontal axis perpendicular to the rod AB. The axis passes through the point C, the mid-point of AB. The centre of mass of the system is G.

(a) Show that $CG = a$. *(2 marks)*

(b) Show that the moment of inertia of the system about the axis through C is $32ma^2$. *(3 marks)*

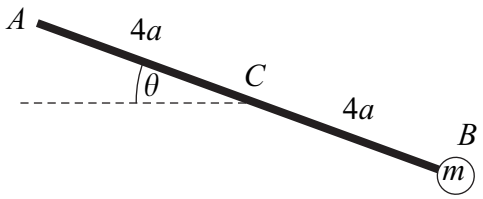
(c) The system is held at rest in a horizontal position, as shown in **Figure 1**.

Figure 1



The system is released from rest and begins to rotate. At time t, the system has turned through an angle θ, as shown in **Figure 2**.

Figure 2



(i) Show that the angular speed of the system at time t is

$$\sqrt{\frac{g\sin\theta}{4a}}$$

(4 marks)

(ii) Find an expression, in terms of a, g and θ, for the angular acceleration of the system. *(2 marks)*

(iii) The component of the reaction force perpendicular to the rod at C is Y.

The component of the reaction force parallel to the rod at C is X.

Find X and Y in terms of m, g and θ. *(6 marks)*

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Copyright © 2013 AQA and its licensors. All rights reserved.